Циркуляры Главной Астрономической Обсерватории в Пулкове

№ 6 Февраль 1933

February 1933

POULKOVO OBSERVATORY CIRCULAR

№ 6

Table of contents.

The Excitation of the Metastabile States in the Gaseous Nebulae.

It is known that the physical conditions in the gaseous nebulae are very favourable for the accumulation of atoms in a metastabile state.

There are two different types of atomic processes which are responsible for the excitation of metastabile states in the gaseous nebulae: the fluorescence phenomenon and the electronic collisions. We shall consider both possibilities:

The fluorescence phenomenon. We shall consider an atom which has three stationary levels 1, 2, 3 with the energies $\varepsilon_1 < \varepsilon_2 < \varepsilon_3$. Let n_1, n_2, n_3 be the number of atoms in cubic centimetre in corresponding levels. The relative values of these numbers in the case of stationary distribution are determined by radiation field and atomic constants (transition probabilites). The conditions of stationarity are indeed:

$$\begin{aligned} B_{12}\left(n_1-\frac{g_1}{g_2}n_2\right)\rho_{12}+B_{13}\left(n_1-\frac{g_1}{g_3}n_3\right)\rho_{13}-n_2\frac{g_1}{g_2}B_{12}\sigma_{12}-n_3\frac{g_1}{g_3}B_{13}\sigma_{13}=0 \\ B_{13}\left(n_1-\frac{g_1}{g_3}n_3\right)\rho_{13}+B_{23}\left(n_2-\frac{g_2}{g_3}n_3\right)\rho_{23}-n_3\left\{\frac{g_1}{g_3}\sigma_{13}B_{13}+\frac{g_2}{g_3}\sigma_{23}B_{23}\right\}=0 \end{aligned} \tag{1}$$

where B_{ik} is Einstein's probability coefficient corresponding to the transition $i \rightarrow k$, g_k is the weight of the k-th level, ρ_{ik} is the density of radiation in the frequency

$$\nu_{ik}=\frac{\varepsilon_k-\varepsilon_i}{h}$$

and

$$\sigma_{ik}=\frac{8\pi h\nu^3_{ik}}{c^3} \tag{2}$$

h, c and π have their usual meaning.

Thus the product

$$A_{ki}=B_{ik}\sigma_{ik}\frac{g_i}{g_k} \tag{3}$$

is the Einstein's probability coefficient of spontaneous transition $k \rightarrow i$.

Write the equations (1) in the form

$$B_{12}\rho_{12}+B_{13}\rho_{13}=\frac{g_1}{g_2}B_{12}(\sigma_{12}+\rho_{12})\frac{n_2}{n_1}+\frac{g_1}{g_3}B_{13}(\sigma_{13}+\rho_{13})\frac{n_3}{n_1}$$
$$B_{13}\rho_{13}=-B_{23}\rho_{23}\frac{n_2}{n_1}+\left[\frac{g_1}{g_3}B_{13}(\sigma_{13}+\rho_{13})+\frac{g_2}{g_3}B_{23}(\sigma_{23}+\rho_{23})\right]\frac{n_3}{n_1} \tag{4}$$

Before solving these equations we shall make some simplifications, corresponding to the physical conditions in nebulae. The radiation density ρ_{ik} may be represented in the form:

$$\rho_{ik}=W\frac{\sigma_{ik}}{e^{\frac{h\nu_{ik}}{kT}}-1} \tag{5}$$

Here T is the surface-temperature of the nucleus and the factor W is defined by the relation:

$$W=\frac{1}{4}\left(\frac{r_*}{r_n}\right)^2 \tag{6}$$

where r_* is the radius of the nucleus and r_n is the distance of the point of nebula under consideration from the nucleus. If W is a small quantity ($W<10^{-3}$) the densities ρ_{ik} in the brackets of (4) may be neglected, compared with σ_{ik} and we have:

$$B_{12}\rho_{12}+B_{13}\rho_{13}=\frac{g_1}{g_2}B_{12}\sigma_{12}\frac{n_2}{n_1}+\frac{g_1}{g_3}B_{13}\sigma_{13}\frac{n_3}{n_1}$$
$$B_{13}\rho_{13}=-B_{23}\rho_{23}\frac{n_2}{n_1}+\left(\frac{g_1}{g_3}B_{13}\sigma_{13}+\frac{g_2}{g_3}B_{23}\sigma_{23}\right)\frac{n_3}{n_1} \tag{7}$$

Solving these equations we obtain:

$$\frac{n_2}{n_1}=\frac{B_{12}\rho_{12}(g_1B_{13}\sigma_{13}+g_2B_{23}\sigma_{23})+g_2B_{13}B_{23}\rho_{13}\sigma_{23}}{\frac{g_1}{g_2}B_{12}\sigma_{12}(g_1B_{13}\sigma_{13}+g_2B_{23}\sigma_{23})+g_1B_{13}B_{23}\sigma_{13}\rho_{23}} \tag{8}$$

We suppose that the second level is a metastabile one, i. e. that the quantity B_{12} is small compared with B_{13} and B_{23}. Therefore the members containing the factor $B_{12}\rho_{12}$ may be neglected compared with the term containing $B_{13}\rho_{13}$. The formula (8) then becomes:

$$\frac{n_2}{n_1}=\frac{g_2B_{13}B_{23}\rho_{13}\sigma_{23}}{\frac{g_1}{g_2}B_{12}\sigma_{12}(g_1B_{13}\sigma_{13}+g_2B_{23}\sigma_{23})+g_1B_{13}B_{23}\sigma_{13}\rho_{23}} \tag{9}$$

We may write

$$\rho_{ik}=W\sigma_{ik}\bar{\rho}_{ik}\quad\text{where}\quad\bar{\rho}_{ik}=\frac{1}{e^{\frac{h\nu_{ik}}{kT}}-1} \tag{10}$$

Then

$$\frac{n_2}{n_1}=\frac{g_2\bar{\rho}_{13}W}{\frac{g_1^2}{g_2}\frac{B_{12}\sigma_{12}}{B_{23}\sigma_{23}}+g_1\frac{B_{12}\sigma_{12}}{B_{13}\sigma_{13}}+g_1\bar{\rho}_{23}W} \tag{11}$$

Neither the transition $3\rightarrow 1$, nor the transition $3\rightarrow 2$ are forbidden. Therefore the quantities B_{13} and B_{23} will be of the same order of magnitude. Therefore the ratio $\frac{B_{12}}{B_{13}}$ and $\frac{B_{12}}{B_{23}}$ are the small quantities of the same order of magnitude.

We shall now consider two possibilites: $W\lessgtr\frac{B_{12}}{B_{13}},\frac{B_{12}}{B_{23}}$.

Case I. $W<\frac{B_{12}}{B_{13}},\frac{B_{12}}{B_{23}}$. In this case the last term in denominator may be neglected ($\bar{\rho}_{23}$ is ordinarily of the order of unity) and we have

$$\frac{n_2}{n_1}=\frac{g_2}{g_1}W\frac{\bar{\rho}_{13}}{\frac{g_1}{g_2}\frac{B_{12}\sigma_{12}}{B_{23}\sigma_{23}}+\frac{B_{12}\sigma_{12}}{B_{13}\sigma_{13}}}$$

In order to estimate the order of magnitude of $\frac{n_2}{n_1}$ we may put: $g_1=g_2$; $B_{23}\sigma_{23}=$ $=B_{13}\sigma_{13}$. We get

$$\frac{n_2}{n_1}\simeq W\frac{B_{13}\sigma_{13}}{2B_{12}\sigma_{12}}\bar{\rho}_{13} \tag{12}$$

If on the other hand the second level is not metastabile (ordinary level) and B_{12} is of the same order of magnitude as B_{13} and B_{23}, we may neglect the last term in (8) and write

$$\left(\frac{n_2}{n_1}\right)_{ord}=\frac{B_{12}\sigma_{12}(g_1B_{13}\sigma_{13}+g_2B_{23}\sigma_{23})W\bar{\rho}_{12}+g_2B_{13}B_{23}\sigma_{13}\sigma_{23}W\bar{\rho}_{13}}{\frac{g_1}{g_2}B_{12}\sigma_{12}(g_1B_{13}\sigma_{13}+g_2B_{23}\sigma_{23})} \tag{13}$$

When estimating the order of magnitude we may put: $g_1=g_2$; $B_{12}\sigma_{12}=B_{13}\sigma_{13}=$ $=B_{23}\sigma_{23}$. Then

$$\left(\frac{n_2}{n_1}\right)=W\bar{\rho}_{12}+\frac{1}{2}W\bar{\rho}_{13} \tag{14}$$

The main difference between (12) and (14) is the presence in (12) of a large factor $\frac{B_{13}\sigma_{13}}{B_{12}\sigma_{12}}$. Therefore we may assert that in the case under consideration the number of atoms in the metastabile state is $\frac{B_{13}\sigma_{13}}{B_{12}\sigma_{12}}$ times larger than in any ordinary excited state.

Case II. $W > \frac{B_{12}}{B_{13}}, \frac{B_{12}}{B_{23}}$. In this case the first two members in the denominator of (11) may be neglected. Therefore we have

$$\frac{n_2}{n_1} = \frac{g_2 \bar{\rho}_{13}}{g_1 \bar{\rho}_{23}} = \frac{g_2}{g_1} \frac{e^{\frac{h\nu_{23}}{kT}} - 1}{e^{\frac{h\nu_{13}}{kT}} - 1} \tag{15}$$

If $h\nu_{13} > kT$ we obtain

$$\frac{n_2}{n_1} = \frac{g_2}{g_1} \left(e^{-\frac{h\nu_{12}}{kT}} - e^{-\frac{h\nu_{13}}{kT}} \right) \tag{16}$$

If at the same time $h\nu_{23} > kT$ and $h\nu_{12} > kT$

$$\frac{n_2}{n_1} = \frac{g_2}{g_1} e^{-\frac{h\nu_{12}}{kT}} \tag{17}$$

In this case the ratio $\frac{n_2}{n_1}$ is approximately defined by Boltzmann's law. This result was already obtained by Rosseland.[1]

The physical meaning of the difference between the two cases considered above is the following: the first two terms in the denominator of (11) correspond to the forbidden transition from the metastabile state to the normal state. The last term in the denominator of (11) corresponds to the transitions from the metastabile state to the higher states. These transitions are stimulated by the corresponding radiation of course. In the Ist case the forbidden transitions are predominant. The forbidden line will appear then in full strength. In the second case the stimulated transitions to the higher levels are predominant, and if W is sufficiently large, the relative number of forbidden transitions will be very small and the forbidden line will disappear. Shortly, in the second case the density of radiation will be large enough to make the collisions of the metastabile atoms with light-quanta sufficiently frequent.

There may be some doubts as to the possibility of application of our formulae to the gaseous nebulae because the photo-electric ionization plays in these nebulae a far more important role than the line excitation. But we may treat the ionized atom as an atom in the energy level with very large weight g_3, and the continuous spectrum behind the head of the principal series of atom as a very wide spectral line. In fact the quantity B_{13} determined in the manner that $n_1 B_{13} \rho_{13}$ is the number of atoms ionized per second will be of the same order of magnitude as the B—coefficients for the first lines of the principal series. We shall only remark that on account of the large optical thickness of nebula in ordinary lines of the principal series the radiation of nucleus in these lines will be absorbed in the inner layers of the nebula and therefore the first member of (14) vanishes while the second

[1] S. Rosseland, Astrophysik.

member remains nearly unchanged since the optical thickness in the continuous spectrum is about 10^4 times smaller than in the ordinary lines of the principal series. Hence

$$\left(\frac{n_2}{n_1}\right)_{ord} = \frac{1}{2} W \rho_{13} = \frac{1}{2} \frac{W}{e^{\frac{h\nu_{13}}{kT}} - 1} \tag{18}$$

when the second level is not a metastabile one.

It seems that the writer's observations[1] are in good agreement with this formula. The expression (18) shows that our assertion that in the case I the number of atoms in the metastabile state is $\frac{B_{13}\sigma_{13}}{B_{12}\sigma_{12}}$ times larger, than in any ordinary excited state must be satisfied more exactly, than we should expect.

Applications to the gaseous nebulae. We have

$$A_{ki} = \frac{g_i}{g_k} B_{ik} \sigma_{ik}$$

For the first lines of each principal series A_{ki} is of the order $10^8 sec^{-1}$ if the corresponding transition is not forbidden. Taking $g_i = g_k$ we obtain for these lines

$$B_{ik} = \frac{10^8}{\sigma_{ik}}$$

As we have mentioned above, the quantity B_{13} corresponding to the bound-free transitions will be of this order of magnitude. In the Ist case it will be

$$W < \frac{B_{12}}{B_{13}} \quad \text{or} \quad W < \frac{B_{12}\sigma_{13}}{10^8}$$

Or introducing $B_{12} = \frac{A_{12}\, g_2}{\sigma_{12}\, g_1}$

$$W < \frac{A_{12}\,\sigma_{13}\, g_2}{10^8\, \sigma_{12}\, g_1} = 10^{-8} \frac{g_2}{g_1} \left(\frac{\nu_{13}}{\nu_{12}}\right)^3.$$

The quantity $\frac{g_2}{g_1}\left(\frac{\nu_{13}}{\nu_{12}}\right)^3$ is usually of the order of unity and we find

$$W < 10^{-8} A_{12}.$$

In the planetaries and diffuse nebulae W is of the order 10^{-14}. Hence

$$\tau_2 = \frac{1}{A_{12}} < 10^6 \text{ sec.}$$

where τ_2 is the mean life-time of the metastabile state.

---

[1] Zeitschrift für Astrophysik, *6*, 107, 1933.

Thus, if the mean life-time of the metastabile state is shorter than a week, the conditions of Case I are fulfilled. Only when the mean life-time of the given state is larger than 10^6 sec. will the formulae of case II be applied. As examples we shall consider the following metastabile levels: the states $2\,S$ of H, $2\,S$ of He^+, $2^1\,S$ of parhelium and the state $2^3\,S$ of orthohelium. The first three of these are metastabile because the only possible transition of the type

$$2S \rightarrow 1S$$

is „forbidden" as a transition from one even state to another even state. The last state $2^3\,S$ of orthohelium is metastabile because the only possible transition of the type

$$2^3\,S - 1^1\,S$$

is forbidden not only as a transition from one even state to another but also as an intercombination between an orthohelium and a parhelium levels. The metastability of $2^3\,S$ of He will be therefore of a higher degree than the metastability of the first three levels.

If we suppose that the mean life-time for the first three types is of the order of 1 sec. or 10 sec. i. e. of the same order as the mean life-time of the levels corresponding to the „nebulium" radiation the formulae of case I will be applicable. The ratio $\frac{n_2}{n_1}$ will be for these states 10^8 or 10^9 times larger than the same ratio for ordinary lines.[1]

Only for the level $2^3\,S$ of He may we expect such a long mean life-time that the case II may occur. A very large proportion of He atom will be then in the state $2^3\,S$ and in favourable conditions a considerable optical depth of the nebula in principal series may arise.

The progress in our knowledge about the mean life-time of the different metastabile states will essentialy extend the possibilities of the investigations of the excitation in the gaseous nebulae.

Application to the Wolf-Rayet stars. To determine which of our two cases is realised in the gaseous shell surrounding a Wolf-Rayet star the knowledge of W is required. We have no data about this subject but it seems that W will be scarcely smaller than 10^{-8}, and is perhaps larger. We know, indeed, that during a month after the outburst the Novae develop many features of the Wolf-Rayet Spectrum. Taking the velocity of the expansion of the gaseous shell 1000 km/sec and the radius of the star after the rejection of gases 10^6 km we obtain for W at the end of

[1] Observations have shown that the number of excited atoms in the ordinary excited states of hydrogen is of the order 10^4 per square centimetre of the nebular disc. The number of hydrogen atoms in the state $2\,S$ will be therefore 10^{12} or 10^{13} per square centimetre and the optical thickness of the nebula in the first two lines of Balmes series may reach 0.1 or 1.

the month the value $0.5 \cdot 10^{-7}$. For such value of W the formula (16) will be applicable to the levels with mean life-time longer than 10^{-1} sec. Some of the metastabile states will have longer mean life-time. Such is in the first line undoubtedly the state 2^3S of orthohelium. The accumulation of atoms in this state may cause considerable optical depth in the lines of the principal series of orthohelium. The number of atoms in the state 2^3S of He per square centimetre of the surface of the envelope may be estimated in the following manner.

Not all quanta capable to ionize the normal He-atom emitted by the central star are absorbed by gaseous envelope, because in the opposite case at the temperatures of the Wolf-Rayet stars the lines of He would be much stronger than the lines of He^+. But we shall suppose that about one per cent of the quanta mentioned will be absorbed, to explain the observed intensities of He-lines. Therefore the optical thickness of the gaseous envelope for the frequencies lying behind the frequency of ionization of normal He will be about 0.01. If the absorption coefficient per each He-atom is of the same order as the absorption coefficient behind the head of the Lyman series of H, the number of normal He-atoms per square centimetre will be therefore about 2.10^{15}. Applying the formula (16) we find that the number of atoms in the state 2^3S of He per each square centimetre of the surface of the envelope will be about 10^{14}. Such number of atoms will produce a considerable optical thickness of the envelope in the lines of the principal series of orthohelium. The violet absorption components of the corresponding Wolf-Rayet bands would then be observable.

The Interstellar Calcium. It is known that the state 3^2D of Ca^+ is metastabile, since the transition $3^2D \rightarrow 4^2S$ is forbidden according to the selection rule about the azimuthal quantum number. But this forbiddance is not as strict as in the cases considered above because in this case there exists a finite probability of transition corresponding to quadrupole radiation. The mean life-time of the state 3^2D will be probably of the order of 10^{-3} sec. The dilution factor is of the order of 10^{-16} — 10^{-17} and the formula (12) will be applicable in our case. Anyhow we shall have

$$\frac{n_2}{n_1} < 10^{-11}$$

and in spite of the metastability of the state 3^2D the number of atoms accumulated in this state will be so small that the radiation of stars will not be absorbed in any considerable degree by these atoms. The absence of the infra-red doublet in the spectra of the B stars showing the detached calcium lines will be a new evidence for the interstellar nature of these lines.

The collisional excitation. We shall now consider an atom which has only two levels 1 and 2 with the energies ε_1 and ε_2. We suppose that the collisions of the second kind may excite some nebular atoms to the metastabile state. The atom may after that pass inthe normal state either spontaneously emitting a quantum of the forbidden line or transmitting the energy of the excitation to a free electron. All other types of the transitions will be neglected. If $b_{12}\,dt$ is the probability of an inelastic collision which excites the normal atom and a_{21} is the probability of the transition of an excited atom in the normal state by means of a superelastic collision the condition of stationarity will have the form:

$$b_{12}n_1 - (A_{21} + a_{21})\,n_2 = 0 \tag{19}$$

When the velocity distribution of electrons obeys the Maxwell's law we have

$$b_{12} = \frac{g_1}{g_2}\,a_{21}\,e^{-\frac{h\nu_{12}}{kT}}$$

(19) becomes

$$\frac{n_2}{n_1} = \frac{g_2}{g_1}\,\frac{a_{21}}{A_{21} + a_{21}}\,e^{-\frac{h\nu_{12}}{kT}}.$$

If $A_{21} > a_{21}$, i. e. if the density of electrons is low, the ratio $\frac{n_2}{n_1}$ is smaller than

$$\frac{g_2}{g_1}\,e^{-\frac{h\nu_{12}}{kT}}.$$

In this case the spontaneous transitions are predominant and the forbidden lines appear in their full strength. If $A_{12} < a_{21}$ the forbidden lines will then be weakened or will disappear altogether.

V. Ambarzumian.